

The Tensor Part of the Skyrme Energy Density Functional

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Abstract

We systematically study the effect of the \mathbf{J}^2 tensor terms in the Skyrme energy functional on properties of spherical nuclei. We build a set of 36 parameterizations covering a wide range of the corresponding parameter space. We analyze the impact of the tensor terms on the evolution of single-particle-level splittings along chains of semi-magic nuclei in spherical calculations. We find that positive values of the coupling constants of proton-neutron and like-particle tensor terms allow for a qualitative description of the evolution of neutron and proton single-particle level splittings in chains of Ca, Ni and Sn isotopes.

Key words: Nuclear energy density functional, tensor interaction, single-particle energies
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The tensor force has been identified very early as an important part of the nucleon-nucleon interaction, and its effects, e.g. the specific correlations it generates and their importance for the binding of nuclear systems, have been studied in infinite and few-body systems. However, it is only recently that energy density functional (EDF) practitioners have renewed their interest in the various “tensor terms” occurring in the nuclear EDF [1,2,3,4,5,6,7]. We attempt in this contribution to sketch a systematic study of the variation of tensor-term parameters and constrain their values [8].

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In spherical symmetry, a zero-range tensor force added on top of the usual Skyrme effective vertex [9] contributes to the EDF through terms proportional to \mathbf{J}^2 [10,11] where \mathbf{J} is the spin-orbit current density vector, here defined through its radial component:

$$J_q(r) = \frac{1}{4\pi r^3} \sum_{n,j,\ell} (2j+1) v_{njl}^2 \left[j(j+1) - \ell(\ell+1) - \frac{3}{4} \right] \psi_{njl}^2(r). \quad (1)$$

The resulting total spin-orbit field for neutrons reads (invert n and p for protons)

$$W_n(r) = \frac{W_0}{2} (2\nabla\rho_n + \nabla\rho_p) + \alpha J_n + \beta J_p, \quad (2)$$

where the first term comes from the zero-range spin-orbit vertex and the two others from the tensor vertex. When the functional is derived from such a Skyrme-tensor vertex the coupling parameters α and β can be chosen independently of the more standard force or functional parameters. In order to study their effects, we build a series of parameterizations, for each of which (α, β) are fixed and all other parameters are fitted according to a protocol [8] similar to the one used for the construction of the Saclay-Lyon parameterizations. They are labelled TIJ , with indices I and J related to α and β through $\alpha = 60(J-2)$ MeV fm⁵ and $\beta = 60(I-2)$ MeV fm⁵.

Tensor terms alter the strength and shape of the spin-orbit potential, Eq. (2), when \mathbf{J} varies due to the filling of a single-particle state. The tensor contribution to $W_q(r)$ thus depends on details of the relative placement of the levels, and is subject to much sharper relative variations than the spin-orbit contribution. As such, it can be constrained by examining the variation of the relative placement of single-particle states in a series of nuclei differing by the filling of levels which significantly contribute to \mathbf{J} .

The first example, displayed on the left panel of Fig. 1, is the tin chain, along which the $h_{11/2}$ neutron level is filled (between $N = 64$ and 82) yielding a large contribution to \mathbf{J}_n and thus to the proton spin-orbit field due to the $\beta \mathbf{J}_n \cdot \mathbf{J}_p$ coupling. This has been previously identified as a possible source of the change of slope (as a function of N) in the spacing of proton $1g_{7/2}$ and $1h_{11/2}$ levels [12]. The spacing of $2d_{5/2}$ and $1g_{7/2}$ levels is affected in a similar way. We can reproduce the magnitude of the single-particle-level spacing shifts by setting the np-coupling β to 120 MeV fm⁵, however, the effect appears to occur at too large a neutron number, owing to the incorrect placement of the neutron $1h_{11/2}$ level relative to the $3s_{1/2}$ and $2d_{3/2}$ ones.

The neutron-neutron coupling can be constrained by examining the spacings of neutron levels in nuclei of the same isotopic chain. In the Ca chain, for example, the filling of the $1f_{7/2}$ level between ^{40}Ca and ^{48}Ca affects the splitting of the neutron $1d$ shell, yielding a shift of the $1d_{3/2}$ relative to the $2s_{1/2}$ one. Similarly, the filling of the $1f_{5/2}$ level between ^{56}Ni and ^{68}Ni acts on the $2p$ and $1f$ states and produces a relative shift of the $1f_{5/2}$ and $2p_{1/2}$ levels. The right panel of Fig. 1 displays the evolution of level splittings related to the latter effects, as a function of the like-particle coupling constant α :

$$\delta^{\text{Ca}} = \left(\varepsilon_{1d_{3/2}}^{^{48}\text{Ca}} - \varepsilon_{2s_{1/2}}^{^{48}\text{Ca}} \right) - \left(\varepsilon_{1d_{3/2}}^{^{40}\text{Ca}} - \varepsilon_{2s_{1/2}}^{^{40}\text{Ca}} \right), \quad (3)$$

$$\delta^{\text{Ni}} = \left(\varepsilon_{1f_{5/2}}^{^{68}\text{Ni}} - \varepsilon_{2p_{1/2}}^{^{68}\text{Ni}} \right) - \left(\varepsilon_{1f_{5/2}}^{^{56}\text{Ni}} - \varepsilon_{2p_{1/2}}^{^{56}\text{Ni}} \right). \quad (4)$$

The two cases are consistent with each other, as a satisfactory comparison to experiment is obtained for values in the range $\alpha \simeq 120 - 150$ MeV fm⁵.

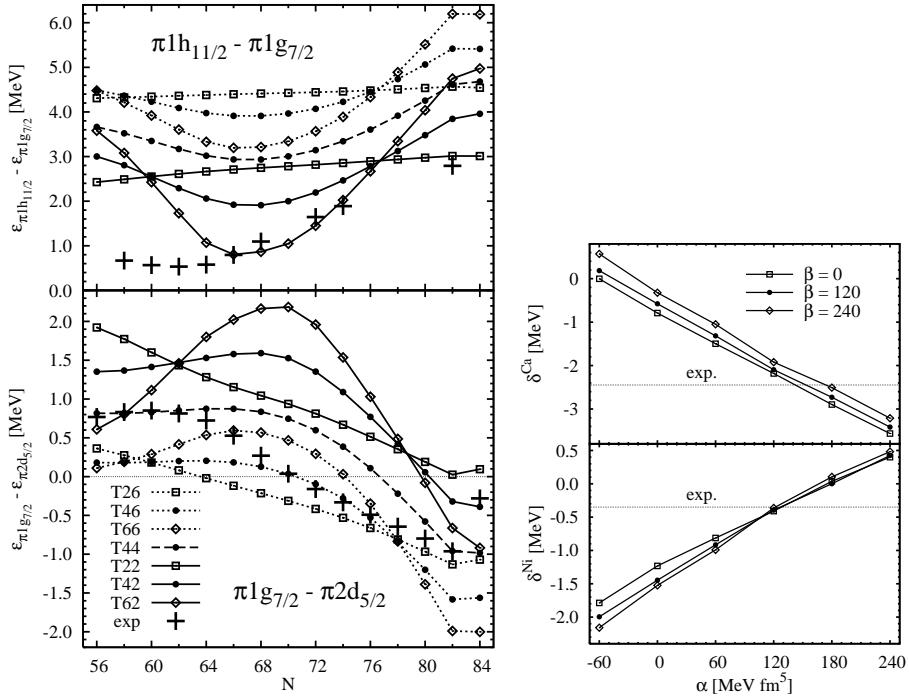


Fig. 1. Incidence of a variation of J^2 coupling constants on single-particle level shifts. Left panel: Distance of the proton $1h_{11/2}$ and $1g_{7/2}$ levels (top) and of the proton $2d_{5/2}$ and $1g_{7/2}$ levels (bottom), for the chain of tin isotopes. Right panel: Shift of the distance between the neutron $1d_{3/2}$ and $2s_{1/2}$ levels when going from ^{40}Ca to ^{48}Ca , Eq. (3) (top) and of the neutron $1f_{5/2}$ and $2p_{1/2}$ levels when going from ^{56}Ni to ^{68}Ni , Eq. (4) (bottom).

To conclude, we have constrained the tensor-term parameters, yielding $\alpha \sim \beta \sim 120 \text{ MeV fm}^5$. However, the addition of these terms does not provide a global improvement of single-particle spectra, as has been expected from more limited studies, and even deteriorates some aspects of single-particle spectra in doubly-magic nuclei [8]. Complementary work on the central and spin-orbit parts should thus be performed.

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